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NANO WEAKLY g# CLOSED MAPS AND NANO WEAKLY g# OPEN MAPS IN NANO TOPOLOGICAL SPACES

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Abstract: The Nano Weakly $g\#$ open set is one of the stronger form of nano topological spaces. In this article we introduce the concept of Nano Weakly $g\#$ open maps and Nano Weakly $g\#$ closed maps in Nano topological spaces and investigate their neighbor maps such as $N\alpha$ open map, $N\alpha g$ open map, $Ng\alpha$ open map, Nsg open map and Ngs open map and their respective nano closed maps in nano topological spaces. Also we analyze some of their related properties.

Keywords and Phrases: $NWg\#$ open set, $NWg\#$ closed set, $NWg\#$ continuous function, $NWg\#$ open map, $NWg\#$ closed map.

2020 Mathematics Subject Classification: 54C05.

1. Introduction

In 2013 Lellis Thivagar and Carmel Richard [4] had introduced nano closed maps, nano open maps and nano homeomorphisms in nano topological spaces. In 2016 Bhuvaneswari. K and Ezhilarasi. A [1] introduced Nsg closed maps, Nsg open maps in nano topological spaces. In 2017 Sathish Mohan. M, Rajendran. V, Devika. A and Vani. R [8] introduced nano semi open maps and nano semi closed maps in nano topological spaces. After that 2020 Mythili Gnanapriya. K and Bhuvaneswari. K [5] introduced Nano g closed maps, Nano g open maps in nano topological spaces. In 2020 Sulochana Devi. P and Bhuvaneswari. K [9] defined Nano regular generalized closed maps in nano topological spaces. This paper, theoretically introduced and investigate a new class of functions called Nano Weakly $g\#$ closed maps and Nano Weakly $g\#$ open maps in nano topological spaces. The properties and relationship between other existing maps are derived with suitable examples and some characterizations are discussed in details.

2. Preliminaries

Definition 2.1. *Let U be a non-empty finite set of objects called the universe and R be an equivalencerelation on U named as indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another the pair (U, R) is said to be approximation space. Let $X \subseteq U$*
1. The Lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$ That is

$$L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\},$$

where $R(X)$ denotes the equivalence class determined by X
2. The Upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$ that is

$$U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\},$$

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.

$$B_R(X) = U_R(X) - L_R(X)$$

Proposition 2.2. *If (U, R) is an approximation space and $X, Y \subseteq U$*
1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\Phi) = U_R(\Phi) = \Phi$ and $L_R(U) = U_R(U) = U$
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$
10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3. *Let U be the universe, R be an equivalence relation on U and*

$\tau_R(X) = \{U, \Phi, L_R(X), U_R(X), B_R(X)\}$ *where* $X \subseteq U$. *Then by Proposition (2.2)* $\tau_R(X)$ *satisfies the following axioms:*
1. U *and* $\Phi \in \tau_R(X)$,
2. The union of elements of any sub collection of $\tau_R(X)$ *is in* $\tau_R(X)$,
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ *is in* $\tau_R(X)$. $\tau_R(X)$ *is a topology on* U *called the Nano topology on* U *with respect to* X. *We call* $(U, \tau_R(X))$ *is the Nano topological space. The elements of* $\tau_R(X)$ *are called as Nano open sets and* $[\tau_R(X)]^c$ *is called as the dual Nano topology of* $\tau_R(X)$.

Remark 2.4. *If* $\tau_R(X)$ *is the Nano topology on* U *with respect to* X *then the set* $B = \{U_R(X), L_R(X), B_R(X)\}$ *is the basis for* $\tau_R(X)$.

Definition 2.5. $(U, \tau_R(X))$ *is a Nano Topological Space with respect to* X *and if* $A \subseteq U$, *then*
(i). The Nano interior of A *is defined as the union of all Nano open subsets of* A *and it is denoted by* $Nint(A)$, *which is the largest Nano open subset of* A.
(ii). The Nano closure of A *is defined as the intersection of all Nano closed sets containing* A *and it is denoted by* $Ncl(A)$, *which is the smallest Nano closed set containing* A.

Definition 2.6. [4] *A map* $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ *is said to be Nano closed map if the image of every nano closed set in* $(U, \tau_R(X))$ *is nano closed set in* $(V, \tau_{R'}(Y))$.

Definition 2.7. [4] *A map* $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ *is said to be Nano open map if the image of every nano open set in* $(U, \tau_R(X))$ *is nano open set in* $(V, \tau_{R'}(Y))$.

3. Nano Weakly $g\#$ Closed Maps in Nano Topological Spaces

Definition 3.1. *Let* $(U, \tau_R(X))$ *and* $(V, \tau_{R'}(Y))$ *be two nano topological spaces. A map* $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ *is said to be Nano Weakly g# closed map (NWg# closed map) if the image of every Nano closed set in* $(U, \tau_R(X))$ *is* $NWg\#$ *closed in* $(V, \tau_{R'}(Y))$.

Example 3.2. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$ then the nano topology $\tau_R(X) = \{U, \Phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. The Nano closed sets are $\tau_{R^c}(X) = \{U, \Phi, \{b, c, d\}, \{a, c\}\{c\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{y\}, \{z, w\}\}$ *and* $Y = \{x, z\}$ then the Nano topology $\tau_{R'}(Y) = \{V, \Phi, \{x\}, \{z, w\}, \{x, z, w\}\}$ the nano closed sets are $\tau c_{R'(Y)} = \{\Phi, V, \{y, z, w\}, \{x, y\}, \{y\}\}$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ by $f(a) = w, f(b) = z, f(c) = y, f(d) = x$. Consider the nano closed sets of $(U, \tau_R(X))$ and their images are

$f(c) = \{y\}, f(a, c) = \{y, w\}, f(b, c, d) = \{x, y, z\}$. The image of every nano closed set in $(U, \tau_R(X))$ is $NWg\#$ closed in $(n, \tau_{R'}(Y))$. Therefore the map f is $NWg\#$ closed map.

Theorem 3.3. *Every nano closed map is $NWg\#$ closed map but not conversely.*
Proof. Let Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces and let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a nano closed map. Every nano closed set A in $(U, \tau_R(X)), f(A)$ is nano closed in $(V, \tau_{R'}(Y))$. Since every nano closed set is $NWg\#$ closed, $f(A)$ is $NWg\#$ closed in $(V, \tau_{R'}(Y))$ for every Nano closed set A in $(U, \tau_R(X))$. Hence the map f is $NWg\#$ closed map.
The converse of the above theorem is not true as seen from the following example.

Example 3.4. In example 3.2 define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ by $f(a) = w, f(b) = z, f(c) = y, f(d) = x$. The map f is $NWg\#$ closed map. Consider the nano closed sets of $(U, \tau_R(X))$. Now $f(a, c) = \{y, w\}$ *and* $f(b, c, d) = \{x, y, z\}$ which are not nano closed in $(V, \tau_{R'}(Y))$. Hence f is not nano closed map.

Theorem 3.5. *Every nano α closed map is $NWg\#$ closed map but not conversely.*
Proof. Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y)))$ be two nano topological spaces and let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a nano α closed map. Every nano closed set A in $(U, \tau_R(X)), f(A)$ is nano α closed in $(V, \tau_{R'}(Y))$. Since every nano α closed set is $NWg\#$ closed , $f(A)$ is $NWg\#$ closed in $(V, \tau_{R'}(Y))$ for every nano closed set A in $(U, \tau_R(X))$. Hence the map f is $NWg\#$ closed map.
The converse of the above theorem is not true as seen from the following example.

Example 3.6. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$ then the Nano topology $\tau_R(X) = \{U, \Phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. The nano closed sets are $\tau_{R^c}(X) = \{U, \Phi, \{b, c, d\}, \{a, c\}, \{c\}\}$. The $NWg\#$ closed sets are $\{U, \Phi, \{b\}, \{c\}, \{d\}, \{b, c\}, \{c, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{y\}, \{z, w\}\}$ and $Y = \{x, z\}$ then the nano topology $\tau_{R'}(Y) = \{V, \Phi, \{x\}, \{z, w\}, \{x, z, w\}\}$. Now $\tau_{R^{c'}(Y)} = \{\Phi, V, \{y, z, w\}, \{x, y\}, \{y\}\}$. The $NWg\#$ closed sets are $\{V, \Phi, \{y\}, \{z\}, \{w\}, \{x, y\}, \{y, z\}, \{y, w\}, \{x, y, z\}, \{y, z, w\}, \{x, y, w\}\}$. The Nano α closed sets are $\{V, \Phi, \{y\}, \{x, y\}, \{y, z, w\}\}$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ by $f(a) = w, f(b) = z, f(c) = y, f(d) = x$. The map f is $NWg\#$ closed map. Now $f(a, c) = \{y, w\}$ and $f(b, c, d) = \{x, y, z\}$ which are not nano α closed set in $(V, \tau_{R'}(Y))$. Hence f is not nano α closed map.

Theorem 3.7. *Every nano regular closed map is $NWg\#$ closed map but not con-*

versely.
The proof is similar to the proof of theorem 3.5.
The converse of the above theorem is not true as seen from the following example.

Example 3.8. In example 3.6 the map f is $NWg\#$ closed map. The nano regular closed sets of $(V, \tau_{R'}(Y))$ are $\{x, y\}, \{y, z, w\}$. Now $f(c) = \{y\}, f(a, c) = \{y, w\}$ and $f(b, c, d) = \{x, y, z\}$ which are not nano regular closed set of $(V, \tau_{R'}(Y))$. Hence the map f is not Nano regular closed map.

Theorem 3.9. *Every nano αg closed map is $NWg\#$ closed map but not conversely.*
Proof. Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces and let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a nano αg closed map. Every nano closed set A in $(U, \tau_R(X)), f(A)$ is nano αg closed set in $(V, \tau_{R'}(Y))$. Every nano αg closed set is $NWg\#$ closed, $f(A)$ is $NWg\#$ closed in $(V, \tau_{R'}(Y))$ for every nano closed set A in $(U, \tau_R(X))$. Hence the map f is $NWg\#$ closed map.

Example 3.10. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$ then the Nano topology $\tau_R(X) = \{U, \Phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. The nano closed sets are $\tau_{R^c}(X) = \{U, \Phi, \{b, c, d\}, \{a, c\}\{c\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{z\}, \{y, w\}\}$ and $Y = \{y, w\}$ then the nano topology $\tau_{R'}(Y) = \{V, \Phi, \{y, w\}\}$ the nano closed sets are $\tau_{R'C}(Y)) = \{\Phi, V, \{x, z\}\}$. The $NWg\#$ closed sets are $\{V, \Phi, \{x\}, \{y\}, \{z\}, \{w\}, \{x, y\}, \{y, z\}, \{z, w\}, \{x, z\}, \{x, w\}, \{x, y, z\}, \{x, z, w\}, \{x, y, w\}, \{y, z, w\}\}$. Nano αg closed sets are $\{V, \Phi, \{x\}, \{z\}, \{x, y\}, \{y, z\}, \{z, w\}, \{x, z\}, \{x, w\}, \{x, y, z\}, \{x, z, w\}, \{x, y, w\}, \{y, z, w\}\}$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ by $f(a) = x, f(b) = y, f(c) = w, f(d) = z$. The map f is $NWg\#$ closed map. Now $f(c) = \{w\}$ which is not nano αg closed sets of $(V, \tau_{R'}(Y))$. Hence f is not Nano αg closed map.

Theorem 3.11. *Every nano $g\alpha$ closed map is $NWg\#$ closed map but not conversely.*
The proof is similar to the proof of theorem 3.9.
The converse of the above theorem is not true as seen from the following example.

Example 3.12. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$ then the Nano topology $\tau_R(X) = \{U, \Phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. The nano closed sets are $\tau_{R^c}(Y)) = \{U, \Phi, \{b, c, d\}, \{a, c\}, \{c\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{z\}, \{y, w\}\}$ and $Y = \{y, w\}$ then the Nano topology $\tau_{R'}(Y) = \{V, \Phi, \{y, w\}\}$. $\tau_{R'C}(Y) = \{\Phi, V, \{x, z\}\}$. $NWg\#$ closed sets are $\{V, \Phi, \{x\}, \{y\}, \{z\}, \{w\}, \{x, y\},$

$\{y,z\},\{z,w\},\{x,z\},\{x,w\},\{x,y,z\},\{x,z,w\},\{x,y,w\},\{y,z,w\}\}$. Nano $g\alpha$ closed sets are $\{V,\Phi,\{x\},\{z\},\{x,z\}\}$. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=y, f(b)=w, f(c)=x, f(d)=z$. The map f is $NWg\#$ closed map. Now $f(a,c)=\{x,y\}$ and $f(b,c,d)=\{x,z,w\}$ are not nano $g\alpha$ closed sets of $(V,\tau_{R'}(Y)))$. Hence f is not nano $g\alpha$ closed map.

Theorem 3.13. *Every nano generalized closed map (Nano g closed map) is $NWg\#$ closed map but not conversely.*

Proof. Let $(U,\tau_R(X))$ and $(V,\tau_{R'}(Y))$ be two nano topological spaces and let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a nano generalized closed map. Thus every nano closed set A in $(U,_{\tau R}(X)), f(A)$ is nano generalized closed set in $(V,\tau_{R'}(Y))$. Since every nano generalized closed set is $NWg\#$ closed, $f(A)$ is $NWg\#$ closed in $(V,\tau_{R'}(Y))$ for every nano closed set A in $(U,\tau_R(X))$. Hence the map f is $NWg\#$ closed map.

The converse of the above theorem is not true as seen from the following example.

Example 3.14. In example 3.12 define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ by $f(a)=z, f(b)=w, f(c)=y, f(d)=x$. The map f is $NWg\#$ closed map. $NWg\#$ closed sets are $\{x\},\{y\},\{z\},\{w\},\{x,y\},\{y,z\},\{z,w\},\{x,z\},\{x,w\},\{x,y,z\},\{x,z,w\},\{x,y,w\},\{y,z,w\}\}$. The nano generalized closed sets are $\{x\},\{z\},\{x,y\},\{y,z\},\{z,w\},\{x,z\},\{x,w\},\{x,y,z\},\{x,z,w\},\{x,y,w\},\{y,z,w\}$. Here $f(c)=\{y\}$ which is not nano generalized closed set of $(V,\tau_{R'}(Y))$. Hence the map f is not nano generalized closed map in nano topological spaces.

Theorem 3.15. *Let $(U,\tau_R(X))$ and $(V,\tau_{R'}(Y))$ be two nano topological spaces and let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ then the following statements are true.*
(i) Every $NWg\#$ closed map is Ngs closed map.
(ii) Every $NWg\#$ closed map is Nsg closed map.
(iii) Every $NWg\#$ closed map is Nano gsp closed map.
(iv) Every $NWg\#$ closed map is Nano β closed map.

Proof. (i) Let $(U,\tau_R(X))$ and $(V,\tau_{R'}(Y))$ be two nano topological spaces and let the map $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a $NWg\#$ closed map. Every nano closed set A in $(U,\tau_R(X)), f(A)$ is nano $NWg\#$ closed in $(V,\tau_{R'}(Y))$. Since every $NWg\#$ closed set is Ngs closed, $f(A)$ is Ngs closed in $(V,\tau_{R'}(Y))$ for every nano closed set A in $(U,\tau_R(X))$. Hence the map f is Ngs closed map.
The proof of (ii) to (iv) are similar to proof of (i).

The converse of the above theorem is not true as seen from the following example.

Example 3.16. Let $U = \{a, b, c, d\}$ with $U/R = \{\{b\}, \{d\}, \{a, c\}\}$ and $X = \{a, b\}$ then the nano topology $\tau_R(X) = \{U, \Phi, \{b\}, \{a, c\}, \{a, b, c\}\}$. The nano closed sets are $\tau_{R^c}(X) = \{\Phi, U, \{d\}, \{b, d\}, \{a, c, d\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{w\}, \{y, z\}\}$ and $Y = \{x, z\}$ then the nano topology $\tau_{R'}(Y) = \{V, \Phi, \{x\}, \{y, z\}, \{x, y, z\}\}$ and $\tau_{R'C}(Y) = \{\Phi, V, \{w\}, \{x, w\}, \{y, z, w\}\}$. The $NWg\#$ closed sets are $\{V, \Phi, \{y\}, \{z\}, \{w\}, \{z, w\}, \{y, w\}, \{x, w\}, \{x, z, w\}, \{x, y, w\}, \{y, z, w\}\}$. Nano gs closed sets are $\{V, \Phi, \{x\}, \{y\}, \{z\}, \{w\}, \{x, w\}, \{y, z\}, \{z, w\}, \{y, w\}, \{x, z, w\}, \{x, y, w\}, \{y, z, w\}\}$. The Nano sg closed sets are $\{V, \Phi, \{x\}, \{y\}, \{z\}, \{w\}, \{x, w\}, \{y, z\}, \{z, w\}, \{y, w\}, \{x, z, w\}, \{x, y, w\}, \{y, z, w\}\}$. The Nano gsp closed sets are $\{V, \Phi, \{x\}, \{y\}, \{z\}, \{w\}, \{x, y\}, \{x, z\}\{x, w\}, \{y, z\}, \{z, w\}, \{y, w\}, \{x, z, w\}, \{x, y, w\}, \{y, z, w\}\}$. The Nano β closed sets are $\{V, \Phi, \{x\}, \{y\}, \{z\}, \{w\}, \{x, y\}, \{x, z\}, \{x, w\}, \{y, z\}, \{z, w\}, \{y, w\}, \{x, z, w\}, \{x, y, w\}, \{y, z, w\}\}$.
Define $f : (U, \tau_R(X))) \to (V, \tau_{R'}(Y))$ by $f(a) = w, f(b) = z, f(c) = x, f(d) = y$. The map f is Ngs closed map, Nsg closed map, Nano gsp closed map and Nano β closed map. Now $f(b, d) = \{y, z\}$ which is not $NWg\#$closed set in $(V, \tau_{R'}(Y))$. Hence the map f is not $NWg\#$ closed map.

Theorem 3.17. *A map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is $NWg\#$ closed map if and only if for each subset of V and for each nano open set G containing $f^{-1}(B)$ there exists a $NWg\#$ open set F of V such that $B \subseteq F$ and $f^{-1}(F) \subseteq G$.*
Proof. Assume that $f : (U, \tau_R(X))) \to (V, \tau_{R'}(Y))$ is $NWg\#$ closed map . By definition 3.1 the image of every nano closed set in $(U, \tau_R(X)))$ is $NWg\#$ closed in $(V, \tau_{R'}(Y))$. Let G be a nano open subset of $(U, \tau_R(X)))$ and let B be a subset of $(V, \tau_{R'}(Y))$ such that $f^{-1}(B) \subseteq G$. Since G is nano open in $(U, \tau_R(X))), U - G$ is nano closed in $(U, \tau_R(X)))$. Also f is $NWg\#$ closed function, $f(U - G)$ is $NWg\#$ closed in $(V, \tau_{R'}(Y))$. Let $F = V - f(U - G)$ is $NWg\#$ open set containing B such that $f^{-1}(F) \subseteq G$.
Conversely assume that for each subset B of $(V, \tau_{R'}(Y))$ and for each nano open set G containing $f^{-1}(B)$ there exists a $NWg\#$ open set F of $(V, \tau_{R'}(Y))$ such that $B \subseteq F$ and $f^{-1}(F) \subseteq G$. Let H be a nano closed subset of $(U, \tau_R(X)))$. Now $U - H$ is nano open subset of $(U, \tau_R(X))$ and $f^{-1}(V - f(H)) \subseteq U - H$. By assumption there exists a $NWg\#$ open set F of $(V, \tau_{R'}(Y))$ such that $V - f(H) \subseteq F$ and $f^{-1}(F) \subseteq U - H$ implies $H \subseteq U - f^{-1}(F)$ and $V - F \subseteq f(H) \subseteq f(U - f^{-1}(F)) \subseteq V - F$ hence $F(H) \subseteq V - F$. Thus $V - F$ is $NWg\#$closed subset of $(V, \tau_{R'}(Y))$. Therefore $f(H)$ is $NWg\#$ closed in $(V, \tau_{R'}(Y))$. The image of every nano closed set in $(U, \tau_R(X))$ is $NWg\#$ closed in $(V, \tau_{R'}(Y))$. Hence f is $NWg\#$ closed map in nano topological spaces.

Theorem 3.18. *If $f : (U, \tau_R(X)) \to (V, \tau_R(Y))$ is nano closed map and*

$g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z)))$ is a $NWg\#$ closed map then their composition $gof : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is $NWg\#$ closed map.

Proof. Assume that f is nano closed map and g is a $NWg\#$ closed map. Let A be a nano closed set in $(U, \tau_R(X))$. Since f is nano closed map, $f(A)$ is nano closed set in $(V, \tau_R(Y))$ for every nano closed set A in $(U, \tau_R(X))$. Since g is $NWg\#$ closed map, $g(f(A))$ is $NWg\#$ closed in $(W, \tau_{R''}(Z))$. Then $gof(A) = g(f(A))$ is $NWg\#$ closed set in$(W, \tau_{R''}(Z))$. Hence gof is $NWg\#$ closed map in nano topological spaces.

Theorem 3.19. *A map $f : (U, \tau_R(X)) \to (V, \tau_R(Y))$ is $NWg\#$ closed map if and only if $NWg\#\ cl(f(A)) \subset f(Ncl(A))$ for every subset A of $(U, \tau_R(X))$.*

Proof. Suppose that f is $NWg\#$ closed set and $A \subset (U, \tau_R(X))$. Then $f(Ncl(A))$ is $NWg\#$ closed map in $(V, \tau_R(Y))$. Also we have $f(A) \subset f(Ncl(A))$. Since A is $NWg\#$ closed if and only if $NWg\#cl(A) = A$ and for any two subsets A and B of $(U, \tau_R(X))$ if $A \subset B$ then $NWg\#cl(A) \subset NWg\#cl(B), NWg\#cl(f(A)) \subset NWg\#cl(f(Ncl(A))) = f(Ncl(A))$.

Conversely let A be any nano closed set of $(U, \tau_R(X))$. Thus $A = Ncl(A)$, now $f(A) = f(Ncl(A))$. By assumption $NWg\#cl(f(A)) \subset f(Ncl(A))$ for every subset A of $(U, \tau_R(X))$. Then $f(A) = f(Ncl(A)) \supset NWg\#cl(f(A))$. Therefore $f(A) = NWg\#cl(f(A))$ that is $f(A)$ is $NWg\#$ closed in $(V, \tau_R(Y))$. Hence the map f is $NWg\#$ closed map.

Theorem 3.20. *Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z)$ be two mappings such that their composition $g\ o\ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z)$ is $NWg\#$ closed . Then the following statements are true*

(i) If f is nano continuous and bijective then g is $NWg\#$ closed map.

(ii) If g is $NWg\#$ irresolute and bijective then f is $NWg\#$ closed map.

Proof. (i) Suppose that f is nano continuous. Let A be nano closed subset of $(V, \tau_{R'}(Y))$. Since f is nano continuous, $f^{-1}(A)$ is nano closed in $(U, \tau_R(X)))$. Since $g\ o\ f$ is $NWg\#$ closed, $g\ o\ f(f^{-1}(A))$ is $NWg\#$ closed in $(W, \tau_{R''}(Z))$. That is $g(A)$ is $NWg\#$ closed in $(W, \tau_{R''}(Z))$. Also f is bijective. Hence g is $NWg\#$ closed map.

(ii) Let B be a nano closed set of $(U, \tau_R(X))$. Since $g\ o\ f$ is $NWg\#$ closed, $g\ o\ f(B)$ is $NWg\#$ closed in $(W, \tau_{R''}(Z)$. Also g is $NWg\#$ irresolute $g^{-1}(g\ o\ f(B))$ is $NWg\#$ closed in $(V, \tau_{R'}(Y))$. That is $f(B)$ is $NWg\#$ closed in $(V, \tau_{R'}(Y))$. Since g is bijective, f is $NWg\#$ closed map.

Remark 3.21. *Let the mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y)))$ be a $NWg\#$ closed map and let $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z)$ be a nano closed map then their composition need not be a $NWg\#$ closed map as it seen from the following example.*

Example 3.22. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{d\}, \{b, c\}\}$ and $X = \{a, c\}$ then the Nano topology $\tau_R(X) = \{U, \Phi, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $\tau_{R^c}(X) = \{U, \Phi, \{b, c, d\}, \{a, d\}, \{d\}\}$. The $NWg\#$ closed sets are $\{\Phi, U, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{d\}, \{c\}, \{b\}\}$. Let $V = \{x, y, z, w\}$ with $V/R^{'}(Y) = \{\{y\}, \{w\}, \{x, z\}\}$ and $Y = \{x, y\}$ then the nano topology $\tau_{R'}(Y) = \{V, \Phi, \{y\}, \{x, z\}, \{x, y, z\}\}$ and $\tau_{R'^c}(Y) = \{\Phi, V, \{x, z, w\}, \{y, w\}, \{w\}\}$. The $NWg\#$ closed sets are $\{V, \Phi, \{x\}, \{w\}, \{z\}, \{x, w\}, \{z, w\}, \{y, w\}, \{x, z, w\}, \{x, y, w\}, \{y, z, w\}$. Define the bijective function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y)$ by $f(a) = x, f(b) = y, f(c) = z$, and $f(d) = w$. Then $f(U) = V, f(\Phi) = \Phi, f(d) = w, f(a, d) = \{x, w\}, f(b, c, d) = \{y, z, w\}$. Thus for every nano closed set in $(U, \tau_R(X))$ its image is $NWg\#$ closed set in $(V, \tau_{R'})(Y)$. Hence f is $NWg\#$ closed map. Let $W = \{p, q, r, s\}$ with $W/R^{''}(Z) = \{\{p\}, \{r\}, \{q, s\}\}$ and $Z = \{p, q\}$ then the nano topology $\tau_{R''}(W) = \{W, \Phi, \{p\}, \{q, s\}, \{p, q, s\}\}$ and $\tau R^{''c}(Z) = \{\Phi, W, \{q, r, s\}, \{p, r\}, \{r\}\}$. The $NWg\#$ closed sets are $\{W, \Phi, \{q\}, \{r\}, \{s\}, \{p, r\}, \{r, s\}, \{q, r\}, \{p, q, r\}, \{p, r, s\}, \{q, r, s\}\}$. Define the bijective function $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ by $g(x) = q, g(y) = p, g(z) = s$, and $g(w) = r$. For every nano closed set in$(V,_{\tau R'}(Y)), g(V) = W, g(\Phi) = \Phi, g(w) = r, g(y, w) = \{p, r\}, g(x, z, w) = \{q, r, s\}$ which are nano closed set in $(W, \tau_{R''}(Z))$. Hence g is nano closed map. Now consider the composition of two maps $(g\ o\ f) : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$. Now $(g\ o\ f)(d) = \{r\}$. But $(g\ o\ f)(a, d) = \{q, r\}\ and\ (g\ o\ f)(b, c, d) = \{p, r, s\}$ which are not $NWg\#$ closed in $(W, \tau_{R''}(Z))$ for the nano closed sets $\{a, d\}$ and $\{b, c, d\}$ in $(U, \tau_R(X))$. Hence $(g\ o\ f)$ need not be $NWg\#$ closed map in nano topological spaces.

4. Nano Weakly $g\#$ Open Maps In Nano Topological Spaces

Definition 4.1. *A map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(X))$ is said to be Nano Weakly $g\#$ open map if the image of every nano open set in $(U, \tau_R(X))$ is $NWg\#$ open in $(V, \tau_{R'}(Y))$.*

Theorem 4.2. *Let $(U, \tau_R(X))and(V, \tau_{R'}(Y))$ be two nano topological spaces and if $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ then every nano open map is $NWg\#$ open map but not conversely.*

Proof. Let $f : (U, \tau_R(X))) \to (V, \tau_{R'}(Y))$ be a nano open map. Every nano open set A in $(U, \tau_R(X))), f(A)$ is nano open in $(V, \tau_{R'}(Y))$. Since every nano open set is $NWg\#$ open. Thus $f(A)$ is $NWg\#$ open set in $(V, \tau_{R'}(Y))$ for every nano open set A in $(U, \tau_R(X))$. Hence the map f is $NWg\#$ open map.

The converse of the above theorem is not true as seen from the following example.

Example 4.3. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$

then the nano topology $\tau_R(X) = \{U, \Phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. The $NWg\#$ open sets are $\{\Phi, U, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, \{a, d\}, \{a, b\}\{b, d\}, \{d\}, \{b\}, \{a\}\}$. Let $V = \{x, y, z, w\}$ with $V/R^{'} = \{\{x\}, \{y\}, \{z, w\}\}$ and $Y = \{x, z\}$ then the Nano topology $\tau_{R'}(Y)) = \{V, \Phi, \{x\}, \{z, w\}, \{x, z, w\}\}$. The $NWg\#$ open sets are $\{V, \Phi, \{z\}, \{x\}, \{w\}, \{z, w\}, \{x, w\}, \{x, z\}, \{x, z, w\}, \{x, y, w\}, \{x, y, z\}\}$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ by $f(a) = w, f(b) = x, f(c) = y, f(d) = z$. The map f is $NWg\#$ open map. But $f(a) = \{w\}, f(b, d) = \{x, z\}$ which are not nano open in $(V, \tau_{R'}(Y))$. Hence f is not nano open map.

Theorem 4.4. *Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y)))$ be two nano topological spaces and if $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ then every nano regular open map is $NWg\#$ open but not conversely.*

Proof. Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a nano regular open map. Every nano open set A in $(U, \tau_R(X)), f(A)$ is Nano regular open in $(V, \tau_{R'}(Y))$. Since every nano regular open set is $NWg\#$ open. Thus $f(A)$ is $NWg\#$ open set in $(V, \tau_{R'}(Y))$ for every nano open set A in $(U, \tau_R(X))$. Hence the map f is $NWg\#$ open map.

The converse of the above theorem is not true it is proved by the following example.

Example 4.5. In example 4.3. The Nano regular open sets of $(V, \tau_{R'}(Y))$ are $\{V, \Phi, \{z, w\}, \{x\}\}$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ by $f(a) = z, f(b) = x, f(c) = y, f(d) = w$. The map f is $NWg\#$ open map. Here $f(a) = \{z\}, f(b, d) = \{x, w\}$ and $f(a, b, d) = \{x, z, w\}$ which are not nano regular open sets of $(V, \tau_{R'}(Y))$. Hence the map f is not nano regular open map.

Theorem 4.6. *Let $(U, \tau_R(X))and(V, \tau_{R'}(Y))$ be nano topological spaces and if $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ then every nano α open map is $NWg\#$ open map but not conversely.*

The proof is similar to the proof of theorem 4.4.

The converse of the above theorem is not true it is proved by the following example.

Example 4.7. In example 4.5. The nano α open sets of $(V, \tau_{R'}(Y))$ are $\{V, \Phi, \{x\}, \{z, w\}, \{x, z, w\}\}$ and the function f is $NWg\#$ open map. But $f(a) = \{z\}, f(b, d) = \{x, w\}$ are not nano α open sets of $(V, \tau_{R'}(Y))$. Therefore the map f is not nano α open map.

Theorem 4.8. *Let $(U, \tau_R(X))$ and $(V, \tau^{R^{'}}(Y))$ be nano topological spaces and if $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ then every nano $g\alpha$ open map is $NWg\#$ open map but not conversely.*

The proof is similar to the proof of theorem 4.4.

The converse of the above theorem is not true it is proved by the following example.

Example 4.9. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$ then the Nano topology $\tau_R(X)) = \{U, \Phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. The $NWg\#$ open sets are $\{\Phi, U, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, \{a, d\}, \{a, b\}, \{b, d\}, \{d\}, \{b\}, \{a\}\}$. Let $V = \{x, y, z, w\}$ with $V/R^{'} = \{\{x\}, \{z\}, \{y, w\}\}$ and $Y = \{y, w\}$ then the Nano topology $\tau_{R'}(Y)) = \{V, \Phi, \{y, w\}\}$. The $NWg\#$ open sets are $\{V, \Phi, \{x\}, \{y\}, \{z\}, \{w\}, \{x, y\}, \{y, z\}, \{z, w\}, \{y, w\}, \{x, w\}, \{x, y, z\}, \{x, z, w\}, \{x, y, w\}, \{y, z, w\}$. Nano $g\alpha$ open sets are $\{V, \Phi, \{x\}, \{y\}, \{z\}, \{w\}, \{x, y\}, \{y, z\}, \{z, w\}, \{y, w\}, \{x, w\}, \{x, y, w\}, \{y, z, w\}\}$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ by $f(a) = z, f(b) = x, f(c) = y, f(d) = w$. The map f is $NWg\#$ open map but $f(a, b, d) = \{x, z, w\}$ which is not nano $g\alpha$ open set of $(V, \tau_{R'}(Y))$. Hence the map f is not Nano $g\alpha$ open map.

Theorem 4.10. *Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces and if $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ then every nano αg open map is $NWg\#$ open map but not conversely.*

The proof is similar to the proof of theorem 4.4.

The converse of the above theorem is not true it is proved by the following example.

Example 4.11. In example 4.9. The nano αg open sets of $(V, \tau_{R'}(Y))$ are $\{V, \Phi, \{x\}, \{y\}, \{z\}, \{w\}, \{x, y\}, \{y, z\}, \{z, w\}, \{y, w\}, \{x, w\}, \{x, y, w\}, \{y, z, w\}\}$. The map f is $NWg\#$ open map but $f(a, b, d) = \{x, z, w\}$ which is not nano αg open set of $(V, \tau_{R'}(Y))$ for the nano open set $\{a, b, d\}$ of $(U, \tau_R(X))$. Hence f is not nano αg open map.

Theorem 4.12. *Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be nano topological spaces and if $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ then every nano generalized open map is $NWg\#$ open map but not conversely.*

The proof is similar to the proof of theorem 4.4.

The converse of the above theorem is not true it is proved by the following example.

Example 4.13. In example 4.9. The nano generalized open sets of $(V, \tau_{R'}(Y))$ are $\{V, \Phi, \{x\}, \{y\}, \{z\}, \{w\}, \{x, y\}, \{y, z\}, \{z, w\}, \{y, w\}, \{x, w\}, \{x, y, w\}, \{y, z, w\}$. The map f is $NWg\#$ open map. Now f $(a, b, d) = \{x, z, w\}$ which is not nano generalized open set of $(V, \tau_{R'}(Y))$ for the nano open set $\{a, b, d\}$ of $(U, \tau_R(X))$. Hence the map f is not Nano generalized open map.

Theorem 4.14. *Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces and* *if* $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ *then every* $NWg\#$ *open map is* Ngs *open map but not conversely.*

Proof. Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be a $NWg\#$ open map. Then every nano open set A in $(U, \tau_R(X)), f(A)$ is $NWg\#$ open in $(V, \tau_{R'}(Y))$. Since every $NWg\#$ open set is Ngs open. Thus $f(A)$ is Ngs open set in $(V, \tau_{R'}(Y))$ for every nano open set A in $(U, \tau_R(X))$. Hence the map f is Ngs open.

The converse of the above theorem is not true proved by the following example.

Example 4.15. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$ then the Nano topology $\tau_R(X) = \{U, \Phi, \{a\}, \{b, d\}, \{a, b, d\}\}$ which are Nano open sets. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{y\}, \{w\}, \{x, z\}\}$ and $Y = \{x, y\}$ then the nano topology $\tau_{R'}(Y) = \{V, \Phi, \{y\}, \{x, z\}, \{x, y, z\}\}$. The $NWg\#$ open sets are $\{V, \Phi, \{x\}, \{y\}, \{z\}, \{x, y\}, \{y, z\}, \{x, z\}, \{x, y, z\}, \{x, y, w\}, \{y, z, w\}\}$. Ngs open sets are $\{V, \Phi, \{x\}, \{y\}, \{z\}, \{x, y\}, \{y, z\}, \{x, z\}, \{y, w\}, \{x, z, w\}, \{y, z, w\}, \{x, y, w\}\}$. Define $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ by $f(a) = x, f(b) = w, f(c) = z, f(d) = y$. The map f is Ngs open map. But $f(b, d) = \{y, w\}$ is not $NWg\#$ open set of $(V, \tau_{R'}(Y))$. Hence the map f is not $NWg\#$ open map.

Theorem 4.16. *Let* $(U, \tau_R(X))$ *and* $(V, \tau_{R'}(Y))$ *be nano topological spaces and if* $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ *then every* $NWg\#$ *open map is* Nsg *open map but not conversely.*

The proof is similar to the proof of theorem 4.14.

The converse of the above theorem is not true proved by the following example.

Example 4.17. In example 4.15. Nsg open sets are $\{V, \Phi, \{x\}, \{y\}, \{z\}, \{x, y\}, \{y, z\}, \{x, z\}, \{y, w\}, \{x, z, w\}, \{y, z, w\}, \{x, y, w\}\}$. The map f is Nsg open map But $f(b, d) = \{y, w\}$ is not $NWg\#$ open set of $(V, \tau_{R'}(Y))$. Hence the map f is not $NWg\#$ open map.

Theorem 4.18. *Let* $(U, \tau_R(X))$ *and* $(V, \tau_{R'}(Y))$ *be nano topological spaces and if* $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ *then every* $NWg\#$ *open map is* $Ngsp$ *open map but not conversely.*

The proof is similar to the proof of theorem 4.14.

The converse of the above theorem is not true proved by the following example.

Example 4.19. In example 4.15. The nano gsp open sets are $\{V, \Phi, \{x\}, \{y\}, \{z\}, \{x, y\}, \{y, z\}, \{z, w\}, \{y, w\}, \{x, w\}, \{x, z\}, \{x, y, z\}, \{x, z, w\}, \{x, y, w\}, \{y, z, w\}\}$. The map f is $Ngsp$ open map. But $f\ (b, d) = \{y, w\}$ is not $NWg\#$ open set of $(V, \tau_{R'}(Y))$ for the nano open set $\{b, d\}$ of $(U, \tau_R(X))$. Hence the map f is not $NWg\#$ open map.

Theorem 4.20. *Let* $(U, \tau_R(X))$ *and* $(V, \tau_{R'}(Y))$ *be nano topological spaces and if* $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ *then every* $NWg\#$ *open map is* $N\beta$ *open map but not conversely.*
The proof is similar to the proof of theorem 4.14.
The converse of the above theorem is not true proved by the following example.

Example 4.21. In example 4.13. The $N\beta$ open sets are $\{V, \Phi, \{x\}, \{y\}, \{z\}, \{x, y\}, \{y, z\}, \{z, w\}, \{y, w\}, \{x, w\}, \{x, z\}, \{x, y, z\}, \{x, z, w\}, \{x, y, w\}, \{y, z, w\}\}$. The map f is $N\beta$ open map. But $f(b, d) = \{y, w\}$ is not $NWg\#$ open in $(V, \tau_{R'}(Y))$ for the nano open set $\{b, d\}$ of $(U, \tau_R(X))$. Hence the map f is not $NWg\#$ open map.

Theorem 4.22. *For any bijective function* $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ *the following statements are equivalent.*
$(i) f^{-1} : (V, \tau_{R'}(Y)) \to (U, \tau_R(X))$ *is* $NWg\#$ *continuous.*
(ii) f *is* $NWg\#$ *open map.*
(iii) f *is* $NWg\#$ *closed map.*
Proof. $(i) \Rightarrow (ii)$ Assume that $f^{-1} : (V, \tau_{R'}(Y)) \to (U, \tau_R(X))$ is $NWg\#$ continuous and let K be a nano open set of $(U, \tau_R(X))$.By assumption $(f^{-1})^{-1}(K) = f(K)$ is $NWg\#$ open set in $(V, \tau_{R'}(Y))$. Hence f is $NWg\#$ open map.
$(ii) \Rightarrow (iii)$ Let G be a nano closed set of $(U, \tau_R(X))$. Then G^c is nano open in $(U, \tau_R(X))$. By assumption $f(G^c)$ is $NWg\#$ open set in $(V, \tau_{R'}(Y))$. Then $f(G^c) = (f(G))^c$ is $NWg\#$ open set in $(V, \tau_{R'}(Y))$. Therefore $f(G)$ is $NWg\#$ closed set in $(V, \tau_{R'}(Y))$. Hence f is $NWg\#$ closed map.
$(iii) \Rightarrow (i)$ Let F be a nano closed set in $(U, \tau_R(X))$. By assumption $f(F)$ is $NWg\#$ closed in$(V, \tau_{R'}(Y))$. But $f(F) = (f^{-1})^{-1}(F)$. Therefore f^{-1} is $NWg\#$ continuous on $(V, \tau_{R'}(Y))$.

5. Conclusion

The nano open sets and nano closed sets play a very prominent role in nano topology and its applications. Indeed, a real significant theme in nano topology. The importance of nano topological spaces rapidly increases in both the pure and applied mathematics. In this paper we introduced and investigate the notions of new classes of maps Nano Weakly $g\#$ open maps and Nano Weakly $g\#$ closed maps in nano topological spaces. The relation with their neighbor maps were discussed. Furthermore, the work was extended as Nano weakly $g\#$ homeomorphisms in nano topological spaces.

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